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# Boost covariant gluon distributions in large nuclei

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## Abstract

It has been shown recently that there exist analytical solutions of the Yang–Mills equations for non–Abelian Weizsäcker–Williams fields which describe the distribution of gluons in large nuclei at small  $x$ . These solutions however depend on the color charge distribution at large rapidities. We here construct a model of the color charge distribution of partons in the fragmentation region and use it to compute the boost covariant momentum distributions of wee gluons. The phenomenological applications of our results are discussed.

# 1 Introduction

Recently, we constructed a QCD based model for the distribution of gluons in large nuclei at small  $x$  [1]. These fields are the non-Abelian analog of the well known Weizsäcker–Williams fields in QED. A path integral was written down for the wee partons in the presence of valence quark sources. These sources are static charges on the light cone. In subsequent papers the saddle point solution of the path integral and the influence of quantum fluctuations about the saddle point solution were investigated [7]. It was shown later by one of us in Ref. [2] (with J. Jalilian–Marian, A. Kovner and H. Weigert) that the validity of the original model could be extended to finite nuclei and even protons at very small values of  $x$ . Remarkably, it was also shown by J. Jalilian–Marian et al.[2] and by Kovchegov [3] that the classical Yang–Mills equations for the non-Abelian Weizsäcker–Williams fields can be solved and an analytical solution obtained for the nuclear gluon distribution at small values of  $x$ .

However, the gluon distributions at a particular value of  $x$  depend on the color charge squared per unit area,  $\chi(y, Q^2)$ , which is the integral of the color charge squared at all rapidities greater than the rapidity of interest. Formally, we define  $\chi$  to be

$$\chi(\eta, Q^2) = \int_{\eta}^{\infty} d\eta' \mu^2(\eta', Q^2), \quad (1)$$

where  $\mu^2(\eta', Q^2)$  is the color charge squared per unit area per unit rapidity at the rapidity  $\eta'$  and  $Q^2 >> \Lambda_{QCD}^2$  is the momentum transfer squared of an external probe. Note also that the space–time rapidity is defined by the relation

$$\eta = \eta_R + \log(x_R^-/x^-). \quad (2)$$

Here  $x_R^- \approx 2R/\sqrt{2}\gamma$  is the Lorentz contracted thickness of the nucleus in the infinite momentum frame ( $\gamma >> 1$ ) and  $x^- = (t - z)/\sqrt{2}$  is the longitudinal light cone co-

ordinate. At central rapidities, the space–time rapidity is approximately equal to the momentum space rapidity.

In order to explicitly compute the parton distributions in nuclei at small  $x$ , we need to model the color charge distribution of partons in the fragmentation region. In this paper we will construct a simple model which captures the essential physics of the fragmentation region in nuclei. The expression for  $\chi$  that results will lead to a boost covariant expression for the gluon distribution in large nuclei at small  $x$ . A conceptual issue that is resolved as a consequence is the description of the non–Abelian Weizsäcker–Williams fields in the nuclear rest frame. This enables us in principle to compare our results to calculations in the nuclear rest frame [4].

In large part, this work is an extension of the nice analysis done by Kovchegov [3] who, in a simplified model of a nucleus, formulated in the nuclear rest frame, argued that the nucleus acts as a source of color charge. Here we incorporate a model of the longitudinal dynamics and construct a solution in the fragmentation region of the nucleus. We argue that in most of the fragmentation region for a sufficiently large nucleus, our weak coupling methods are valid.

We should warn the reader, that our results for realistically sized nuclei are semi-quantitative and qualitative at best. For realistically sized nuclei, the typical energy scale in the problem is of the order of 300–400 MeV at RHIC energies and 750–900 MeV at LHC energies at central rapidities [5], which is marginally large enough (for our weak coupling calculations to be valid) only in the case of the latter. In order to get a scale well in the weak coupling regime would require a nuclear baryon number a couple of orders of magnitude larger than that of the largest nucleus or equivalently, much higher energies. Nevertheless, we believe the insight we gather from this picture may be useful for obtaining a reasonable, QCD motivated conceptual framework in which to think about high energy nuclear collisions.

Our results also enable us to study the systematics of nuclear gluon shadowing and other phenomenological observables at low  $x$ . These will be especially relevant for the deep inelastic scattering experiments off nuclei which have been proposed at HERA [6]. A caveat is that our result for the gluon distribution is classical. However, another important result of Jalilian–Marian et al. [2] is that the effective action for wee partons demonstrates a self–similar structure as we go to lower rapidities. The effective color charge  $\chi$  obeys a renormalization group equation analogous to the DGLAP or BFKL equations [8] (we note that in a recent paper, it has been shown explicitly that the BFKL kernel is recovered in this approach [10]). Therefore, at each rapidity slice, the gluon distribution is given by the classical gluon distribution with the quantum effects absorbed in the effective scale  $\chi$  at that rapidity slice. The residual quantum corrections are of order  $\alpha_S \Delta\eta$  which is much smaller than unity for the rapidity slice of width  $\Delta\eta$ . These can in principle be computed. In other words, the gluon distributions computed using our model for  $\chi$ , provide the initial conditions for the renormalization group equations which can be solved self–consistently for  $\chi$ .

The rest of the paper is organized as follows. In section 2, we briefly discuss and summarize the results of Ref. [2] for the classical gluon distribution. In section 3, we construct our model for  $\chi$ , the color charge squared per unit area. Our model depends on one free parameter,  $\kappa$ , which is the average color charge squared per *nucleon* integrated over one unit of rapidity. Section 4 contains a discussion of how  $\kappa$  may be determined and the implications of our model for shadowing in very large nuclei. The final section contains a brief summary and points out directions for future work.

## 2 An analytical solution for correlation functions of non-Abelian Weizsäcker-Williams fields

In Ref. [1, 2], a path integral was written down for wee parton modes in a nucleus with  $x \ll A^{-1/3}$  and transverse momentum  $\Lambda_{QCD} \ll k_t \ll \sqrt{\chi}$ . Here  $\chi = \int_{\eta}^{\infty} d\eta' \mu^2(\eta', Q^2)$ , where  $\mu^2$  is the average color charge squared per unit area at a particular rapidity  $\eta'$  and a transverse momentum resolution  $Q^2$ . The latter is normally associated with the momentum resolution of an external probe. The measure for these modes (in light cone gauge  $A^+ = 0$ ) is

$$Z = \int [dA_t dA^-] [d\rho] \exp \left( iS_{QCD} - ig \int d^4x A^-(x) \rho(x_t, \eta) - \int_0^{\infty} d\eta \int d^2x_t \frac{1}{2\mu^2(\eta, Q^2)} \rho^2(\eta, x_t) \right). \quad (3)$$

Above,  $y$  has the same definition as in Eq. 2. In the trace that defines the path integral, the sum over the valence quark color charges has been replaced by a Gaussian measure with the weight  $\mu^2$ . The action  $S_{QCD}$  above refers only to the pure gauge degrees of freedom. The above action does not include sea quarks (which are  $\alpha_S$  suppressed) but is easily extended to do so. Since  $\mu^2$  is the only scale in the problem, the coupling constant will run as a function of this scale. For  $\mu^2 \gg \Lambda_{QCD}^2$ , our effective theory will be in the regime of weak coupling.

This procedure is justified when the following two assumptions apply<sup>1</sup>. The first is that the valence quarks are static charges on the light cone with  $J^{+,a} = \rho^a(x_t) \delta(x^-)$  as the only large component of the valence quark current. All other components of the current are suppressed by  $1/P^+$ . The second assumption is that the wee partons

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<sup>1</sup>For a recent discussion of a gauge invariant form for the above action, see Ref. [10]. This discussion while of general interest, is not applicable here since we restrict ourselves to the semi-classical result.

couple to a large color charge which is classical. What this implies is that the charge squared per unit area,  $\langle Q^2 \rangle$ , that the wee partons couple to is given by the relation  $\langle Q^2 \rangle = N_q \langle Q^2 \rangle_{quark}$ , where  $N_q$  is the number of quarks per unit area ( $\propto A^{1/3}$ ) and  $\langle Q^2 \rangle_{quark}$  is the charge squared per unit area due to a single quark. It can be shown that the central limit theorem can be applied to this problem and that the corrections to the classical result (which may in principle be computed) are of the order  $1/\sqrt{N_q} \sim A^{-1/6}$  for the root mean square classical charge [9]. For an alternative derivation of the Gaussian measure in a specific model, see Ref. [3].

To compute the classical nuclear gluon distribution function

$$\frac{dN}{d^3k} = \frac{2}{(2\pi)^3} 2|k^+| \int d^3x d^3x' e^{ik \cdot (x-x')} \langle A_i^a(x^-, x_t) A_j^b(x'^-, x'_t) \rangle_\rho , \quad (4)$$

one needs to compute the saddle point solution of Eq. 3, or equivalently, solve the classical Yang–Mills equations for the source  $J^+$  defined above and finally average over the product of the classical fields at two space–time points with the Gaussian measure in Eq. 3. Note that the brackets  $\langle \dots \rangle_\rho$  above correspond to the Gaussian averaging over the static, light cone sources. One finds a particular solution where  $A^\pm = 0$  and  $A^i (i = 1, 2)$  is a pure gauge field which satisfies the equation

$$D_i \frac{dA^i}{d\eta} = g\rho(\eta, x_\perp) , \quad (5)$$

where  $D_i$  is the covariant derivative [1].

In Ref. [2], the above equations were solved to obtain  $A^i$  as a function of the color charge density  $\rho$  and the distribution function computed. Here we will merely state the result of Ref. [2] and refer the reader to that paper for the technical details. The distribution function for the non–Abelian Weizsäcker–Williams fields in coordinate space is

$$\frac{dN}{d^2x_t d\eta} = \frac{4}{g^2} \frac{(N_c^2 - 1)}{N_c x_t^2} \left( 1 - \left( x_t^2 \Lambda_{QCD}^2 \right)^{\frac{q^4 N_c}{16\pi} \chi(\eta, Q^2) x_t^2} \right) . \quad (6)$$

The momentum space distributions are easily obtained by Fourier transforming Eq. 6. The result is

$$\frac{1}{\pi R^2} \frac{dN}{dx_A dk_t^2} = \frac{4}{\pi^2} \frac{(N_c^2 - 1)}{N_c} (r_0 m_N)^2 \frac{1}{4\pi\alpha_S} x_A \int_0^{\log(1/x_A)} dy' \exp(2y') \times \int_0^{1/\Lambda_{QCD}} \frac{du_\perp}{u_\perp} J_0(k_t u_\perp) [1 - (u_\perp \Lambda_{QCD})^\beta]. \quad (7)$$

Above,  $r_0 = 1.12$  fm,  $m_N$  is the nucleon mass and  $\beta = 2\pi\alpha_S^2 N_c \chi(y', Q^2) u_\perp^2$ . For  $\alpha_S \sqrt{\chi} \ll k_t$ , the distribution has the familiar Weizsäcker–Williams form  $\propto g^2/k_t^2$ , while for  $\Lambda_{QCD} \ll k_t \ll \alpha_S \sqrt{\chi}$ , the distribution is  $\propto \frac{1}{g^2} \log(k_t^2/g^4 N_c \chi(y))$ . Even for rapidities far from the beam rapidity, it is reasonable to assume that  $y_{\text{spacetime}} \sim y_{\text{mom}}$  for wee glue and light sea quarks. For  $k_T$  very small compared to  $\alpha_S \sqrt{\chi}$ , the above formula may break down, but we expect that a large number of physical quantities will not be infrared sensitive.

While the above expressions provide a very interesting picture of the behaviour of the gluon distribution as a function of the transverse distance  $x_t$ , the dependence of the distributions on the spacetime rapidity is hidden in the function  $\chi(\eta, Q^2)$  which has not been specified. In order to specify  $\chi$  for all rapidities  $\eta$ , we need a model of the nuclear color charge distribution (at large  $Q^2$ ) in the fragmentation region as well. In the next section, we construct a model for  $\chi$  at some reference scale  $Q_0^2 \gg \Lambda_{QCD}$ . As we will discuss below,  $\chi$  obeys evolution equations in both  $y$  and  $Q^2$ . The important point to note here is that with our model of  $\chi$  the expression in Eq. 6 is then boost covariant. This enables us to describe what the wee parton distributions look like in any frame.

Finally, the parton distributions computed in the above described manner can be related to the empirically extracted gluon distributions at the scale  $Q^2$  by the equation

$$G(x, Q^2) = \int_0^{Q^2} dk_t^2 \frac{dN}{dx dk_t^2}. \quad (8)$$

This simple relation is obtained using light cone quantization and light cone gauge  $A^+ = 0$  [14].

### 3 The rapidity distribution of color charge in nuclei at a fixed resolution scale $Q_0^2$

In this section we shall formulate a model for  $\chi$ , the color charge squared per unit area per unit rapidity, integrated upto the rapidity of interest. A similar model for  $\chi$ , albeit without the explicit  $y$  dependence, was discussed recently by one of us and M. Gyulassy [5]. A detailed model in the fragmentation region is relevant for studying the renormalization group evolution of  $\chi$  towards smaller rapidities and larger  $Q^2$ .

Consider a nucleus at rest. For a large nucleus, the color charge squared per unit longitudinal distance  $z$ ,  $dC/dzd^2x_\perp$  is a constant within the nucleus when measured on longitudinal resolution scales which are large compared to the inter-nucleon separation. The color charge squared is the typical value of charge fluctuations in the fragmentation region of the proton—about 1 unit of rapidity. Here we must include the fluctuations due to the gluons as well as the quarks, so this may be roughly a factor of two larger than is the case for just the valence quarks. Note also that we get a non zero color charge only because we are choosing to measure the charge density on a transverse size scale much less than a fermi. In a large nucleus, the color charge resolved by the probe corresponds to a higher dimensional representation of the color algebra and is therefore classical [1, 9]. The probe sees only small transverse bits of each nucleon but the fluctuation fields add incoherently from nucleon to nucleon.

The non-Abelian Weizsäcker–Williams fields appear as solutions in the nuclear rest frame to an external probe in the following sense. Outside each nucleon, for the

long wavelength small  $x$  modes, there is only the gauge transform of the vacuum field. As we pass from one side of a nucleon to the other, we get a different gauge transform of the vacuum. The field inside a nucleon is not known. Further, since the field is pure gauge outside each nucleon, or in other words, the field strengths are zero, the classical force exerted by the long wavelength modes of one nucleon on a neighboring one is negligible. We therefore have a model of the nucleus with short range nuclear forces where it is the short wavelength ( $x \geq 0.1$ , where  $x \sim 0.1$  is the wavelength corresponding to the inter-nucleon separation) modes which are responsible for intra-nuclear dynamics such as nuclear binding. This model is therefore consistent with the conventional picture that nucleonic effects in nuclei (such as the EMC effect for instance) are unaffected by the physics of small  $x$  modes in the nucleon (for a discussion see for instance Ref. [12]). The typical change in the Weizsäcker–Williams field seen by the external probe should then be described by a stochastic source with a strength which must be determined phenomenologically by an empirical knowledge of the nucleon structure functions. It is with this idea in mind that we formulate the model below.

In the nuclear rest frame we choose to localize the color charge as

$$\frac{d\tilde{C}}{dz} \propto \theta(z + 2R)\theta(-z), \quad (9)$$

where  $\tilde{C}$  is the *classical* color charge squared per unit transverse area at the scale  $Q_0^2$  of the external probe. In the boosted frame, recall that we define  $\eta = \eta_R + \ln(x_R^-/x^-)$  where  $x^- = (t - z)/\sqrt{2}$  and  $x_R^- \approx 2R/\sqrt{2}\gamma$ .

It is easily seen that the above charge distribution is independent of the light cone time  $x^+$  for  $x^+ \ll \gamma$  and  $\gamma \gg 1$ . This result is obtained by boosting the field in the rest frame to the infinite momentum frame. In the classical limit we are working in, the field in one frame is just the Lorentz transform of the field in another. In this limit, the commutators of the color charges vanish, thereby allowing us to relate the

charge distributions in the two frames.

The color charge squared per unit transverse area per unit rapidity in the nuclear rest frame is

$$\frac{d\tilde{C}}{d\eta} = \frac{d\tilde{C}}{dz} \frac{dz}{d\eta}, \quad (10)$$

and from our definition in Eq. 9, we have

$$\frac{dz}{d\eta} = 2Re^{-\eta}, \quad (11)$$

for  $0 < \eta < \infty$ .

The color charge squared density in the nuclear rest frame is

$$\frac{d\tilde{C}}{dz} = \kappa\rho_0 \quad (12)$$

where  $\kappa$  is the typical charge squared *per nucleon* in one unit of rapidity and  $\rho_0$  is the density of nucleons. The latter may be chosen for a large nucleus to have the typical value for nuclear matter density. All of the uncertainty about the color charge distribution *inside each nucleon* is therefore represented in  $\kappa$ .

We can now boost all this to the frame where the nucleus moves very fast. The main effect of the boost is to replace the density  $\rho_0$  by the nuclear gamma factor times  $\rho_0$ . We find therefore that the new charge squared per unit area per unit rapidity (which is the  $\mu^2$  introduced in the previous section) becomes

$$\begin{aligned} \mu^2(\eta, Q_0^2) &= 2R\kappa\rho_0 e^{\eta_R - \eta} ; \eta \geq \eta_R \\ &= 0 ; \eta < \eta_R. \end{aligned} \quad (13)$$

This rapidity distribution is amusing. In the rapidity variable, most of the color charge squared is concentrated in one unit of rapidity above the beam rapidity. This is a boost invariant statement. The region where the charge squared distribution falls

to that typical of a single nucleon is however  $\Delta\eta \sim \ln(R/1 \text{ fm})$ . Even though most of the nucleons are concentrated in one unit of rapidity, it takes the order of  $\ln(R)$  units in rapidity before the charge squared has fallen to a small value!

It is also interesting to consider how this picture of the fragmentation region can be formulated in terms of the momentum space rapidity. A similar argument to the one above applies if we formulate our discussion in terms of a momentum space rapidity that we define as  $y = y_R + \log(k^+/P_R^+)$ . Note that this definition is not the usual one since  $P_R^+$  is *not* the nuclear momentum per nucleon  $P^+$ , but instead the momentum that is conjugate to the Lorentz contracted width  $x_R^-$  of the nucleus. This definition has the following advantages. Firstly, from the uncertainty principle, it is equivalent to the space-time rapidity we defined previously since  $\log(k^+/P_R^+) = \log(x_R^-/x^-)$ . Secondly, by this definition, momenta  $P_R^+ < k^+ < P^+$  correspond to fragmentation region rapidities, or in terms of  $x^-$ , wavelengths shorter than the Lorentz contracted width of the nucleus. One therefore expects that the charge distribution in terms of the above defined momentum space rapidity is identical to the distribution in Eq. 13:  $\chi(\eta, Q^2) \equiv \chi(y, Q^2)$ . What fraction  $x_R$  of the nuclear momentum  $P^+$  does  $P_R^+$  correspond to? It may be determined by requiring that it correspond to a wavelength equal to the Lorentz contracted width of the nucleus  $1/P_R^+ = x_R^-$ . Hence,

$$\frac{1}{x_R P^+} \sim \frac{\sqrt{2}R}{P^+/m_N} \implies x_R \sim A^{-1/3}/(\sqrt{2}r_0 m_N), \quad (14)$$

where  $m_N$  is the nucleon mass and  $r_0 = 1.12 \text{ fm}$ . For  $A = 200$ , this corresponds to  $x_R \sim 0.025$ . Therefore, by our definition, for  $A = 200$ , fragmentation region rapidities  $\tilde{y} > \tilde{y}_R$  correspond to light cone momentum fractions  $x > 0.025$ . Of course even for these large nuclei our classical considerations are very marginal and can be safely applied in the theoretical limit of much larger nuclei or equivalently, when the parton densities in the fragmentation region are already large.

Returning to Eq. 13, because most of the color charge squared is contained in one

unit of rapidity, there is not much dynamical evolution of the glue distribution due to the glue from different nuclei interacting in the fragmentation region. This is because the typical interaction strength is on the order of  $\alpha_S$  which is small due to the high field density and because the typical distance it takes to evolve the gluon distribution due to the interactions of the gluons among themselves is of order  $1/\alpha_S$ . Treating the nucleons as a source of glue in the nuclear fragmentation region is therefore justified so long as sources due to gluons of order one unit of rapidity from the beam rapidity are added to those from the valence quarks. This of course introduces a phenomenological parameter ( $\kappa$  above) which we cannot precisely determine from first principles. However, as will be discussed below  $\kappa$  can be determined empirically from the nucleon gluon distributions.

The gluon rapidity distribution varies slowly ( $\propto \alpha_S$  per unit rapidity) when  $\Delta y \ll 1/\alpha_S$  away from the beam rapidity. It is obtained by solving the classical equations of motion for the source distribution above. Integrating the expression in Eq. 13 above, we obtain

$$\begin{aligned}\chi(y, Q_0^2) &= 2 R \rho_0 \kappa; y < y_R \\ &= 2 R \rho_0 \kappa e^{y_R - y}; y > y_R,\end{aligned}\tag{15}$$

where, we remind the reader,  $y_R$  is the momentum space rapidity defined previously. The ratio of the nuclear gluon distribution to the nucleon parton distribution times A in the nuclear fragmentation region for very large nuclei is plotted in Figure 1. Our classical considerations result therefore in a charge distribution that is independent of rapidity for  $y < y_R$ . Since any gauge invariant configuration of fields must satisfy Gauss's law, the contribution of glue at small x is necessarily associated with the valence degrees of freedom and therefore also exhibits boost invariance. This is also clear from examining Eq. 7.

## 4 Discussion

From our elementary considerations above, we have obtained an analytic, boost covariant form for the color charge squared distribution  $\chi$  in the nuclear fragmentation region at the momentum resolution scale  $Q_0^2$ . It relies on one parameter  $\kappa$  which encapsulates our uncertainty about the non-perturbative physics in the fragmentation region. However, since  $\kappa$  has the transparent physical interpretation of being the color charge squared per nucleon, some of the uncertainty can be eliminated by expressing  $\kappa$  in terms of the nucleon gluon density. Hence, at some reference scale  $Q_0^2$ ,

$$\kappa = \frac{1}{2N_c} \int_{\Delta y \sim 1} dx \left( V(x, Q_0^2) + S(x, Q_0^2) \right) + \frac{N_c}{N_c^2 - 1} \int_{\Delta y \sim 1} dx G(x, Q_0^2). \quad (16)$$

Here  $V$ ,  $S$  and  $G$  denote respectively the valence, sea quark and gluon densities. They are integrated over the roughly one unit of rapidity that one estimates the nucleons to occupy in the fragmentation region. The factors multiplying the parentheses above are the color charge squared per quark and the color charge squared per gluon respectively. The last term for example, has the interpretation that it is the color charge squared per gluon times the number of gluons in the nucleon contained in one unit of rapidity in the fragmentation region. Similarly so for the first term.

The parton densities at the reference scale  $Q_0^2$  can be parametrized by fits to the nucleon deep inelastic scattering data. A convenient one is the NLO parametrization of Glück, Reya and Vogt [15]. In principle, the sensitivity of the results to different parametrizations can be investigated. As is well known by now, the GRV parametrizations of the data at the (extremely low!) reference scale  $(Q_0^{grv})^2 = 0.34 \text{ GeV}^2$ , evolved with the next to leading order Altarelli–Parisi evolution equations, agree well with

the DIS data from HERA. The GRV parametrizations have the generic form

$$xf(x, Q^2) = Nx^a \left(1 + Ax^b + Bx + Cx^{3/2}\right) (1 - x)^D, \quad (17)$$

where  $f = V, S, G$  and the coefficients and powers in the above expression are polynomials of  $\log(\log(Q^2/\Lambda_{QCD}^2))$ . The detailed expressions are given in Ref. [15]. For our classical considerations to apply, we require the reference scale  $Q_0 \gg Q_0^{grv} \sim \Lambda_{QCD}$ . The parton densities which determine  $\kappa$  can then be parametrized by the GRV parton distribution evolved to the scale of interest  $Q_0$ .

With the initial data for  $\chi$  as input, the Wilson renormalization arguments of Ref. [2] can be used to determine it at higher  $Q^2$  and smaller values of  $y$ . It has the form

$$\frac{d\chi}{dydQ^2} = \frac{N_c}{N_c^2 - 1} \frac{1}{\pi R^2} \frac{dN}{dydQ^2}, \quad (18)$$

where  $dN/dydQ^2$  is the distribution function in Eq. 7. This renormalization group equation can be formulated in DGLAP form by first integrating over  $y$ ,

$$\frac{d\chi}{dQ^2} = \frac{N_c}{N_c^2 - 1} \frac{1}{\pi R^2} \int_y^\infty dy' \frac{dN}{dy'dQ^2}. \quad (19)$$

Integrating over  $Q^2$ , we obtain the BFKL-like form

$$\frac{d\chi}{dy} = \frac{N_c}{N_c^2 - 1} \frac{1}{\pi R^2} \int_0^{Q^2} dQ'^2 \frac{dN}{dydQ'^2}. \quad (20)$$

It was shown recently that for large momenta the kernel of the equation satisfied by the right hand side of the above is precisely the BFKL kernel [10]. However in general, as discussed earlier, the gluon distribution on the right hand side of the above equations has a non-linear dependence on  $\chi$ .

A clear picture of shadowing then emerges in the above model. From the perturbative QCD standpoint, one of the possible sources of shadowing is from the the input

parton distributions which are modified upon  $Q^2$  evolution by the splitting functions a la Altarelli–Parisi [16, 20]. The other source is due to recombination and screening effects which enter into the evolution equations as higher twist effects [17, 18]. There have been a number treatments of shadowing which take both these effects into account [19, 20, 21]. It is often believed that current data can be accounted for by shadowing of the initial (non-perturbative) parton distributions and leading twist DGLAP evolution of these distributions.

On the other hand, our model in the previous section suggests that for sufficiently large nuclei i) the initial parton distributions can be determined perturbatively (modulo  $\kappa$ ) and ii) they are not shadowed. However the shadowing away from the fragmentation region is large due to higher twist effects which, in this model, are already present at the classical level. If we substitute the classical result of Eq. 7 into the right hand side of Eq. 19

$$\frac{d\chi}{dQ^2} = \frac{1}{\pi^2 \alpha_S} \int_x^1 \frac{dx'}{x'} \int_0^{\frac{1}{\Lambda_{QCD}}} \frac{dz}{z} J_0(Q z) [1 - \exp(\beta \log(z \Lambda_{QCD}))], \quad (21)$$

where  $\beta = 2\pi\alpha_S^2 N_c \chi(x', Q^2) z^2$ . If we expand out the  $1 - \exp(\dots)$  term on the right hand side and make the identification  $\chi = \frac{N_c}{N_c^2 - 1} x G(x, Q^2)/\pi R^2$ , we obtain an evolution equation in the gluon density  $G(x, Q^2)$  which on the right hand side includes all powers of  $G(x, Q^2)/Q^2 R^2$ —or all twists. The first term in the expansion is just the usual DGLAP evolution equation at small  $x$ . A very similar expression to the above has been obtained by Levin and collaborators working in the nuclear rest frame—and is the perturbative QCD form of the Glauber–Gribov–Mueller formula of high energy scattering [22]. Since the second term in the expansion above can be related to diffractive scattering, diffractive scattering data can be used to determine the degree of nuclear shadowing in this approach. This was done recently by Capella et al. who used HERA diffractive data on e–p scattering to reproduce shadowing data from fixed target experiments—thereby providing a nice check of the Glauber–Gribov–

Mueller expression upto second order in the gluon density [23].

Ultimately, when parton densities are large, the quantum corrections to the above formula will be large and have to be computed. For instance, the second term in the above equation gives only the recombination (or diffractive) contribution to shadowing (with positive sign!). The screening corrections will appear when one includes quantum corrections. If the well known AGK sum rules [24] apply, then as argued by Mueller and Qiu [18]—the expression will be the same as the above but with a negative sign! This has to be confirmed in our approach before anything quantitatively new can be added to current phenomenology. Moreover, we hope that in this effective action approach it will be easier to compute higher order terms as well [25].

## 5 Summary and Outlook

In this paper we have formulated a simple model for  $\chi(y, Q_0^2)$ , the color charge squared integrated from the rapidity of interest upto the beam rapidity at some reference scale  $Q_0^2$ . At this scale,  $\chi$  is the color charge in the nuclear fragmentation region.  $\chi$  depends on the non-perturbative physics at this scale through  $\kappa$  which is the charge squared per nucleon and is simply related to the *nucleon* parton distributions. The expression derived for  $\chi$  is boost covariant. The Wilson renormalization group equation discussed in Ref. [2] can be used to evolve  $\chi$  to  $Q^2 > Q_0^2$  and/or rapidities away from the fragmentation region. We have argued that this classical picture provides important insight into the nature of perturbative gluon shadowing and have argued in the previous section that our results are closely related to similar approaches in the nuclear rest frame. This is indeed as it should be because the model we have constructed is boost covariant. The boost covariant gluon distributions thereby constructed can be applied to a wide range of phenomenology including gluon and

quark shadowing [11, 13], intrinsic quark production [26] and nuclear collisions at ultrarelativistic energies [27, 5]. It is however important to understand the quantum corrections in the infinite momentum frame effective action picture before significant additions can be made to current phenomenology [25].

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## 6 Figure caption

**Fig. 1:** Ratio  $R_A^{frag} = xG_A(x, Q_0^2)/A\kappa$  (where  $\kappa$  is defined as in Eq. 16) as a function of rapidity in the fragmentation region. Above  $y_R$  is defined by Eq. 14;  $y_{beam} - y_R \sim \log(A^{1/3})$  and  $Q_0^2 >> \Lambda_{QCD}^2$ .

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